

Homework 2

Due: April 28

Question 1

Consider a consumer with two goods. Let the price vector be $p = (p_1, p_2)$ and income be y . The indirect utility function is

$$v = \frac{y}{\frac{p_1}{\alpha} + \frac{p_2}{\beta}}$$

- (a) Derive the expenditure function.

- (b) Derive the Hicksian demand function.

- (c) Derive the Marshallian demand function.

(d) For good 1, verify the Slutsky equation:

$$\frac{\partial x_1(p, y)}{\partial p_1} = \frac{\partial h_1(p, u)}{\partial p_1} - \frac{\partial x_1(p, y)}{\partial y} \cdot x_1(p, y)$$

where $u = v(p, y)$.

(e) Find the corresponding direct utility function.

Question 2

Consider a risk-averse individual with wealth w but facing probability π of losing D dollars. An insurance policy is available that pays money in the event of the loss, at a price of $\$q$ with certainty per $\$1$ of payout. Thus, if α units of insurance are bought, the wealth of the individual will be $w - \alpha q$ if there is no loss and $w - \alpha q - D + \alpha$ if the loss occurs.

(a) Suppose $q = \pi$. What is the optimal value of α (i.e. how much insurance should be purchased)? Which state (with or without loss) will the individual prefer?

(b) How does your answer change if $q > \pi$?

(c) Suppose once again $q = \pi$. Now suppose that the individual has a chance to buy a report that will forecast - before the decision to buy insurance - whether the loss will occur. The report has a probability p of providing the correct answer and $(1 - p)$ of giving "no answer" (so there is no chance of a false report). Assume now, additionally, that the insurance company has a rule that nobody can insure for more than D . Derive an expression for the sure payment the individual would be willing to pay to purchase such a report.

Question 3

A firm has production function

$$f(z_1, z_2) = z_1^{1/2}(z_2 - 1)^{1/2}$$

The prices of the inputs are r_1 and r_2 .

(a) Find MP_1 , MP_2 , and MRTS.

(b) If z_2 is fixed at 5, what is the short-run cost function? Find the short-run marginal cost and average cost.

(c) What are the long-run input demand functions? What is the long-run cost function? Find the long-run marginal cost and average cost.

(d) Does the production function exhibit increasing, constant or decreasing returns to scale.

Question 4

The value of non-market goods. Consider a farmer whose production function is $y = f(x, q)$, where x is a vector of market inputs (seed, fertilizer, labor, machinery, etc.) and q is a non-market quality variable. q could be the natural fertility of the soil. It could also be the health of the natural environment (availability of bees and other pollinators; water quality; air quality) or similar variable. The price of output is fixed at p and the prices of the market inputs are w .

(a) Derive a simple expression for the marginal profit of the non-market good q .

(b) Derive an alternative expression for the value of q based on a market input serving as a substitute for q (*Hint: the expression here should be related to MRTS*).

(c) What is the economic interpretation for your results in part (b)?